

Higher Spins as Rolling Tachyons in Open String Field Theory

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Abstract

We find a simple analytic solution in open string field theory which, in the on-shell limit, generates a tower of higher spin vertex operators in bosonic string theory. The solution is related to irregular off-shell vertex operators for Gaiotto states. The wavefunctions for the irregular vertex operators are described by equations following from the cubic effective action for generalized rolling tachyons, indicating that the evolution from flat to collective higher-spin background in string field theory occurs according to cosmological pattern. We discuss the relation between nonlocalities of the rolling tachyon action and those of higher spin interactions.

1 Introduction

Higher spin fields in AdS space-time are known to be a crucial ingredient of *AdS/CFT* correspondence, as most of the composite operators on the conformal field theory (CFT) side are holographically dual to the higher spin modes. Perhaps the best known example of such a correspondence is the one between the higher derivative gauge invariant operators in the $O(N)$ vector model and the symmetric higher spin frame-like fields in AdS_4 [2, 3, 4, 5, 6, 7]. But generically, any operator on the CFT side carrying multiple space-time tensor indices, is expected to be dual to some higher-spin field with mixed symmetry. On the other hand, the higher spin holography implies that any correlator in boundary CFT is reproduced by the worldsheet correlators of vertex operators in string theory, i.e. any gauge-invariant

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observable on the CFT side has its dual vertex operator in AdS string theory. For simple some CFT/gauge theory observables such a correspondence is straightforward. For example, the string counterpart of $Tr(F^2)$ in super Yang-Mills theory is the vertex operator V_ϕ of a dilaton in string theory, while the stress-energy tensor T_{mn} corresponds to the graviton's vertex operator V_{mn} , polarized along the AdS boundary. One problem with checking this conjecture on the string theory side is that we know little about AdS string dynamics beyond semiclassical approximation. This is related to the fact that the first-quantized string theory is background-dependent.

In addition, the higher spin interactions (at least beyond the quartic order) are known to be highly nonlocal, while the standard low-energy effective actions, stemming from vanishing β -function constraints in the first-quantized theories are typically local. This altogether suggests that the second-quantized formalism of the string field theory [1] may be a more adequate formalism to approach the higher spin holography from the string theory side, especially given the formal similarity of background-independent string field theory (SFT) equations of motion [1], and Vasiliev's equations in the unfolding formalism [8, 9, 10, 11]. This naturally poses a question of how the vertex operator to CFT observable correspondence may be extended off-shell, in particular involving the higher spin modes. This does not seem to be obvious. For example, consider a composite operator given by the N 'th power of the stress-energy tensor in CFT:

$$T^n \sim T_{m_1 n_1} \dots T_{m_N n_N} \quad (1.1)$$

In the gravity limit, this operator must be a dual of a certain field of spin $2N$ in AdS with mixed symmetries. But what is the vertex operator description of such an object in string theory? To answer this question, one has to take the colliding limit of N graviton vertex operators in string theory. Taking such a limit does not look simple and must not be confused with the normal ordering. Instead, in order to reproduce the correlation functions correctly in such a limit, one has to retain *all* the terms, up to *all* orders of the operator product expansion (OPE), as the operators are colliding at the common point. Such a limit is well-known in the matrix model formulations of Liouville and Toda theories and plays an important role in extending the AGT conjecture to Argyres-Douglas type of supersymmetric gauge theories with asymptotic freedom. The result is given by rank $N - 1$ irregular Gaiotto-BMT (Bonelli-Maruyoshi-Tanzini) states [14, 12, 13, 20, 21, 17, 15]. These states extend the context of the primary operators in CFT and lead to special representations of Virasoro algebra, being the simultaneous eigenstates of $N + 1$ Virasoro generators:

$$\begin{aligned} L_n |U_N\rangle &= \rho_n |U_N\rangle \quad (N \leq n \leq 2N) \\ L_n |U_N\rangle &= 0 \quad (n > 2N) \end{aligned} \quad (1.2)$$

In the previous work [17] (see also [18, 19] with the related issues addressed) it was shown that the irregular states admit the following irregular vertex operator representation in terms of Liouville or Toda fields :

$$|U_N \rangle = U_N |0 \rangle$$

$$U_N = e^{\vec{\alpha}_0 \vec{\phi} + \sum_{k=1}^N \vec{\alpha}_k \partial^k \vec{\phi}} \quad (1.3)$$

where $\vec{\phi} = \phi_1, \dots, \phi_D$ is either D -component Toda field or (in the context of the present paper) parametrize the coordinates of D -dimensional target space in bosonic string theory. The $\vec{\alpha}_k$ parameters are related to the Virasoro eigenvalues (1.2) according to [17]

$$\rho_n = -\frac{1}{2} \sum_{k_1, k_2; k_1+k_2=n} \vec{\alpha}_{k_1} \vec{\alpha}_{k_2} \quad (1.4)$$

In case of $D \geq 2$, the irregular states, apart from being eigenvalues of positive Virasoro generators, are also the eigenstates of positive modes $W_n^{(p)}$ of the W_n -algebra currents ($3 \leq n \leq D+1$) where

$$W_n^{(p)} = \oint \frac{dz}{2i\pi} z^{p+n-1} W_n(z) \quad (1.5)$$

where W_n are the spin n primaries and $(n-1)N \leq p \leq nN$. so that

$$W_n^{(p)} U_N = \rho_n^p U_N \quad (1.6)$$

and ρ_n^p are degree n polynomials in the components of $\vec{\alpha}$. Note that, while the maximal possible rank n is always at least $D+1$, for higher dimensions ($D > 5$) it is also possible to have the higher ranks $n > D+1$ as well. In general case, the upper bound on n is in fact related to a rather complex problem in the partition theory. Namely, the maximal rank is given by the maximal number n_{max} for which the inequality

$$\sum_{k=1}^{n_{max}} \frac{(k+D-1)! \kappa(n_{max}|k)}{k!} - \sum_{q=1}^{n_{max}-1} \sum_{k=1}^q \frac{(k+D-1)! \kappa(n_{max}|k)}{k!} - (D-1)! \geq 0 \quad (1.7)$$

where $\kappa(n|k)$ is the number of ordered partitions of n with the length k : $n = p_1 + \dots + p_k$; $0 < p_1 \leq p_2 \leq \dots \leq p_k$.

The objects (1.3) are obviously not in the BRST cohomology and are off-shell (except for the regular case $\vec{\alpha}_k = 0$; $k \neq 0$ but make a complete sense in background-independent open string field theory. On the other hand, the U_N -vertices are related to the onshell vertex operators for the higher-spin fields. That is, U_N is the generating vertex for the higher-spin operators through

$$V_{h.s.} = \sum_{s, \{k_1, \dots, k_s\}} H^{\mu_1 \dots \mu_s}(\vec{\alpha}_0) \frac{\partial^s (c U_N)}{\partial \alpha_{k_1}^{\mu_1} \dots \partial \alpha_{k_s}^{\mu_s}} \Big|_{\vec{\alpha}_k=0; k \neq 0} \quad (1.8)$$

where c is the c -ghost, $H^{\mu_1 \dots \mu_s}(\vec{\alpha}_0)$ are the higher spin s fields in the target space with masses $m = \sqrt{2(k_1 + \dots + k_s - 1)}$, at the momentum $\vec{\alpha}_0$ with all the due on-shell constraints on H to ensure the BRST-invariance. Thus the correlation functions (irregular conformal blocks) of the U_N -vertices particularly encode the information about the higher-spin interactions in string theory. At nonzero $\vec{\alpha}_k$ the U_N vertices generate the off-shell extensions of higher-spin wavefunctions, which can be studied using the string field theory techniques. A question of particular interest, studied in this work, is to find the higher spin wavefunction configurations in terms of irregular vertex operators, solving SFT equations of motion analytically. In case if all $\vec{\alpha}_k = 0$, except for $\vec{\alpha}_0$, the U_N vertex becomes a tachyonic primary. Then, multiplied by the space-time tachyon's wavefunction $T(\vec{\alpha}_0)$ $\Psi = c \int d^D \alpha_0 T(\vec{\alpha}_0) U_N$ is an elementary solution of string field theory equations: $Q\Psi + \Psi \star \Psi = 0$ provided that T satisfies the vanishing tachyon's β -function constraints $\beta_T = (\frac{1}{2}\alpha_0^2 - 1)T + \text{const} \times T^2 = 0$ in the leading order of string perturbation theory. This solution is elementary as it describes the *perturbative* background change by a tachyon. In the case of $\vec{\alpha}_k \neq 0$ things become far more interesting. The wavefunction in the string field $\Psi = c \int d^D \vec{\alpha}_0 \dots d^D \vec{\alpha}_N T(\vec{\alpha}_0, \dots, \vec{\alpha}_N) U_N$ can now be regarded as a generating wavefunction for higher spin excitations in string field theory with the SFT solution constraints on T now related to nonperturbative background change due to higher spin excitations and the effective action on T holding the keys to higher spin interactions at all orders, just like the well-known Schnabl's analytic solution [28] describes the physics around the minimum of nonperturbative tachyon potential (that would be calculated up to all orders, from the string perturbation theory point of view) The rest of this letter is organized as follows. In the Section 2 we explore the CFT properties of irregular vertex operators, including the behavior under finite conformal transformations, relevant to the SFT equations of motion. In the present work, we particularly concentrate on the rank 1 and search for the SFT analytic solutions in the form:

$$\Psi = c \int d^D \vec{\alpha} \int d^D \vec{\beta} T(\vec{\alpha}, \vec{\beta}) e^{\vec{\alpha} \vec{\phi} + \vec{\beta} \partial \vec{\phi}} \quad (1.9)$$

We find that, in the leading order, Ψ is an analytic solution if T satisfies the constraints, given by equations of motion described by the nonlocal effective action for generalized rolling tachyons. The nonlocality structures are controlled by the SFT worldsheet correlators and by the conformal transformations of the irregular blocks. The solution in particular provides a nice example of how the star product in string field theory translates into the Moyal product in the analytic SFT solutions. Although we explicitly concentrate on rank one case in this letter, the same structure appears to persist for higher irregular ranks as well. In the discussion section, we investigate the physical meaning and significance of the solution, found in this work. In particular, we relate the nonlocalities in the noncommutative rolling tachyon structures to those of interacting higher-spin theories, as in the context of our calculation $T(\vec{\alpha}, \vec{\beta})$ is the simplest generating function for collective higher-spin excitations and the nonlocalities in the effective action are naturally translated into those for the interactions of higher spins. The effective equations of motion for $T(\vec{\alpha}, \vec{\beta})$ constitute the nonperturbative generalization of β -function equations in perturbative string theory, and possibly can be related to nonlocal

field redefinitions for Vasiliev's invariant functionals [6]. We also comment on connections between the $\vec{\beta}$ -coordinate of the solution to T-duality and the doubling of the space-time and comment on possible relations to double field theory formalism.

2 Irregular Vertices as String Field Theory Solutions: Rank 1 Case

We start with the transformation properties of the irregular vertices under the conformal transformations $z \rightarrow f(z)$, necessary to compute the correlators in string field theory. Straightforward application of the stress tensor to the rank one irregular vertex gives infinitesimal conformal transformation:

$$\begin{aligned} \delta_\epsilon U_1(\vec{\alpha}, \vec{\beta}) &= [\oint \frac{dw}{2i\pi} \epsilon(w) T(w); U_1(\vec{\alpha}, \vec{\beta}, z)] \\ &= \left\{ \frac{1}{12} \partial^3 \epsilon \beta^2 + \frac{1}{2} \partial^2 \epsilon (\vec{\alpha} \vec{\beta}) + \partial \epsilon \left(\frac{1}{2} \alpha^2 + \vec{\beta} \frac{\partial}{\partial \vec{\beta}} \right) + \epsilon \partial_z \right\} U_1(\vec{\alpha}, \vec{\beta}, z) \end{aligned} \quad (2.1)$$

It is not difficult to obtain the finite transformations for U_1 , by integrating the infinitesimal transformations (2.1).

$$U_1(\vec{\alpha}, \vec{\beta}, z) = e^{\vec{\alpha} \vec{\phi} + \vec{\beta} \partial \vec{\phi}} \rightarrow \left(\frac{df}{dz} \right)^{\frac{\alpha^2}{2}} e^{\vec{\alpha} \vec{\phi} + \frac{df}{dz} \vec{\beta} \partial \vec{\phi} + (\vec{\alpha} \vec{\beta}) \frac{d}{dz} \log \left(\frac{df}{dz} \right) + \frac{1}{12} S(f; z)} \quad (2.2)$$

where $S(f; z)$ is the Schwarzian derivative.

It is straightforward to generalize this result to transformation laws for the irregular vertices of arbitrary ranks. For the arbitrary rank N the BRST and finite conformal transformations for the irregular vertices have the form:

$$\begin{aligned} \{Q, cU_N\} &= \left\{ \oint \frac{dz}{2i\pi} (cT - bc\partial c); c e^{i \sum_{q=1}^N \vec{\alpha}_q \partial^q \vec{\phi}} \right\} \\ &= \frac{1}{2} \sum_{q_1=0}^N \sum_{q_2=0}^N \frac{q_1! q_2!}{(q_1 + q_2 + 1)!} (\vec{\alpha}_{q_1} \vec{\alpha}_{q_2}) : \partial^{q_1+q_2+1} c c U_N \\ &\quad + i \sum_{q=1}^N \sum_{p=1}^{q-1} \frac{q!}{p!(q-p)!} \partial^{q-p} c c (\vec{\alpha}_q \frac{\partial}{\partial \vec{\alpha}_{p+1}}) U_N \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} U_N \rightarrow \left(\frac{df}{dz} \right)^{\frac{\alpha_0^2}{2}} e^{-\sum_{q_1, q_2=1}^N S_{q_1|q_2}(f; z) + i \sum_{q=2}^N \sum_{k=1}^{q-1} \sum_{l=1}^k \frac{(q-1)!}{k!(q-1-k)!} \frac{d^{q-k} f}{dz^{q-k}} B_{k|l}(\partial f \dots \partial^{k-l+1} f)(\vec{\alpha}_q \partial^{l+1} \vec{\phi})} \\ \times e^{\partial^n f(\vec{\alpha}_q \partial \vec{\phi})} \end{aligned} \quad (2.4)$$

where $S_{q_1|q_2}(f; z)$ are the generalized Schwarzian derivatives of the rank $q_1 + q_2$, given by

$$S_{q_1|q_2}(f; z) = \frac{1}{(q_1 + q_2)!} B^{(q_1+q_2)}\left(\frac{d}{dz} \log \frac{df}{dz}; \dots \frac{d^{q_1+q_2}}{dz^{q_1+q_2}} \log \frac{df}{dz}\right) - \frac{q_1 + q_2 + 1}{(q_1 + 1)!(q_2 + 1)!} B^{(q_1)}\left(\frac{d}{dz} \log \frac{df}{dz}; \dots \frac{d^{q_1}}{dz^{q_1}} \log \frac{df}{dz}\right) B^{(q_2)}\left(\frac{d}{dz} \log \frac{df}{dz}; \dots \frac{d^{q_2}}{dz^{q_2}} \log \frac{df}{dz}\right) \quad (2.5)$$

where $B^{(q)}(\partial f \dots \partial^q f)$ are the Bell polynomials in the derivatives of $f(z)$ defined according to

$$B^{(q)}(\partial f \dots \partial^q f) = q! \sum_{k=1}^q \sum_{q|p_1 \dots p_k} \frac{\partial^{p_1} f \dots \partial^{p_k} f}{p_1! \dots p_k! \lambda_{p_1}! \dots \lambda_{p_k}!} \quad (2.6)$$

where $q = p_1 + \dots p_k$; $0 < p_1 \leq p_2 \leq \dots \leq p_k$ are the length k ordered partitions of q and λ_p is the multiplicity of an element p of the partition. Given the transformation rules for the irregular vertices, it is now straightforward to compute the correlators relevant to the open string field theory equations of motion. The evaluation of the kinetic term with the string field of the form (1.9) leads to

$$\begin{aligned} & \langle\langle Q\Psi(0) \star \Psi(0) \rangle\rangle = \langle Q\Psi(0) I \circ \Psi(0) \rangle \\ & = \lim_{w \rightarrow \infty} \int d^D \alpha_1 \int d^D \beta_1 \int d^D \alpha_2 \int d^D \beta_2 T(\vec{\alpha}_1, \vec{\beta}_1) y T(\vec{\alpha}_2, \vec{\beta}_2) w^{\alpha_2^2 - 1} e^{-\frac{\vec{\alpha}_2 \cdot \vec{\beta}_2}{z}} \\ & \quad \times \left[\left(\frac{1}{2} \alpha_1^2 - 1 + \vec{\beta} \frac{\partial}{\partial \vec{\beta}} \right) \langle \partial c c e^{i\vec{\alpha}_1 \vec{\phi} + i\vec{\beta}_1 \partial \vec{\phi}}(0) c e^{i\vec{\alpha}_1 \vec{\phi} + i w^2 \vec{\beta}_1 \partial \vec{\phi}}(w) \rangle \right. \\ & \quad + \vec{\alpha}_1 \vec{\beta}_1 \langle \partial^2 c c e^{i\vec{\alpha}_1 \vec{\phi} + i\vec{\beta}_1 \partial \vec{\phi}}(0) c e^{i\vec{\alpha}_1 \vec{\phi} + i w^2 \vec{\beta}_1 \partial \vec{\phi}}(w) \rangle \\ & \quad \left. + \frac{\beta^2}{12} \langle \partial^3 c c e^{i\vec{\alpha}_1 \vec{\phi} + i\vec{\beta}_1 \partial \vec{\phi}}(0) c e^{i\vec{\alpha}_1 \vec{\phi} + i w^2 \vec{\beta}_1 \partial \vec{\phi}}(w) \rangle \right] \quad (2.7) \end{aligned}$$

First of all, as it is clear from (2.7) this correlator is only well-defined in case if the constraint

$$\vec{\alpha} \vec{\beta} = 0 \quad (2.8)$$

is imposed. Since for regular vertex operators α has a meaning of the momentum, the orthogonality constraint (2.8) particularly implies that the β parameter may be related to the Fourier image of the extra coordinates in space-time in the context of double field theory and T -duality (see the discussion section).

Furthermore, note that since the ghost correlator $\langle \partial^n c c(z_1) c(z_2) \rangle = 0$ for $n > 2$, combined with the constraint (2.8) the only surviving terms in the correlator (2.7) are those proportional to $\sim \partial c c$ in $Q\Psi$. In addition, in the on-shell limit $\alpha_0^2 \rightarrow 2$ the correlators involving the terms $\sim \partial c c \vec{\beta} \frac{\partial}{\partial \vec{\beta}}$ and $\sim \partial^2 c c$ are of the order of $\sim \frac{1}{w}$ and vanish.

Thus the only contributing correlator in the kinetic term gives

$$\begin{aligned}
& \langle\langle Q\Psi(0) \star \Psi(0) \rangle\rangle \\
&= \lim_{w \rightarrow \infty} \int d^D \alpha_1 d^D \beta_1 \int d^D \alpha_2 d^D \beta_2 T(\vec{\alpha}_1, \vec{\beta}_1) T(\vec{\alpha}_2, \vec{\beta}_2) w^{\alpha_2^2 - 1} e^{-\frac{\vec{\alpha}_2 \cdot \vec{\beta}_2}{z}} \\
&\quad \times \left\{ \left(\frac{1}{2} \alpha_1^2 - 1 + \vec{\beta} \frac{\partial}{\partial \vec{\beta}} \right) \langle \partial c c e^{i\vec{\alpha}_1 \vec{\phi} + i\vec{\beta}_1 \partial \vec{\phi}}(0) c e^{i\vec{\alpha}_1 \vec{\phi} + i w^2 \vec{\beta}_1 \partial \vec{\phi}}(w) \rangle \right\} \\
&= \int d^D \alpha d^D \beta \frac{1}{2} (\alpha^2 - 1) e^{\beta^2} T(\vec{\alpha}, \vec{\beta}) T(-\vec{\alpha}, -\vec{\beta}) \tag{2.9}
\end{aligned}$$

where we used the orthogonality constraint (2.8). This concludes the computation of the rank 1 contribution to the kinetic term in the SFT equations of motion. Note that, in the regularity limit $\beta^2 \rightarrow 0$ (coinciding with the on-shell limit for the rank 1 irregular operator), one can expand the exponent so that the kinetic term in the Lagrangian becomes

$$\begin{aligned}
&\sim \int d\alpha d\beta T(-\vec{\alpha}, -\vec{\beta}) \left(\frac{1}{2} \alpha^2 + \beta^2 - 1 \right) T(\vec{\alpha}, \vec{\beta}) + \dots \\
&\sim \int dx dy T(x, y) \left(\frac{1}{2} \square_x + \square_y + 1 \right) T(x, y) + \dots \tag{2.10}
\end{aligned}$$

where we skipped the higher derivative terms. The next step is to calculate the cubic terms in the SFT equations. We have:

$$\langle\langle \Psi \star \Psi \star \Psi \rangle\rangle = \int \prod_{j=1}^3 d\alpha_j d\beta_j T(\vec{\alpha}_j, \vec{\beta}_j) \langle g_j^3 \circ c q e^{i\vec{\alpha}_j \vec{\phi} + i\vec{\beta}_j \partial \vec{\phi}}(0) \rangle \tag{2.11}$$

Evaluating the values of g_j^3 and their Schwarzian derivatives at 0 and substituting the transformation laws for Ψ under g_j^3 , as well as the on-shell constraints on $\vec{\alpha}$, it is straightforward

to calculate:

$$\begin{aligned}
\langle\langle \Psi \star \Psi \star \Psi \rangle\rangle &= \int \prod_{j=1}^3 d\alpha_j d\beta_j T(\vec{\alpha}_j, \vec{\beta}_j) = e^{\frac{5}{54}(\beta_1^2 + \beta_2^2 + \beta_3^2)} \left(-\frac{2}{3}\right)^{\frac{1}{2}\alpha_1^2 - 1} \left(-\frac{8}{3}\right)^{\frac{1}{2}\alpha_2^2 + \frac{1}{2}\alpha_3^2 - 2} \\
&\quad \times \langle e^{i\vec{\alpha}_j \vec{\phi} - \frac{2i}{3}\vec{\beta}_j \vec{\partial}\phi}(0) e^{i\vec{\alpha}_j \vec{\phi} - \frac{8i}{3}\vec{\beta}_j \vec{\partial}\phi}(\sqrt{3}) e^{i\vec{\alpha}_j \vec{\phi} - \frac{8i}{3}\vec{\beta}_j \vec{\partial}\phi}(-\sqrt{3}) \rangle \\
&= \int \prod_{j=1}^3 d\alpha_j d\beta_j T(\vec{\alpha}_j, \vec{\beta}_j) \\
&\quad \exp\left\{\frac{5}{54}(\beta_1^2 + \beta_2^2 + \beta_3^2) + \frac{16}{9}(\vec{\beta}_1 \vec{\beta}_2 + \vec{\beta}_1 \vec{\beta}_3 + \vec{\beta}_2 \vec{\beta}_3) + \frac{4}{3\sqrt{3}}(\vec{\alpha}_2 \vec{\beta}_3 \right. \\
&\quad \left. - \vec{\alpha}_3 \vec{\beta}_2) + \frac{2}{3\sqrt{3}}(4\vec{\alpha}_1(\vec{\beta}_3 - \vec{\beta}_2) + \vec{\beta}_1(\vec{\alpha}_3 - \vec{\alpha}_2))\right\} \delta\left(\sum_j \beta_j\right) \delta\left(\sum_j \alpha_j\right) \\
&= \int \prod_{j=1}^2 d\alpha_j d\beta_j T(\vec{\alpha}_1, \vec{\beta}_1) T(\vec{\alpha}_2, \vec{\beta}_2) T(-\vec{\alpha}_1 - \vec{\alpha}_2, -\vec{\beta}_1 - \vec{\beta}_2) \\
&\quad \exp\left\{-\frac{43}{27}(\vec{\beta}_2 \vec{\beta}_3 + \beta_2^2 + \beta_3^2) + \frac{2}{\sqrt{3}}(\vec{\alpha}_2 \vec{\beta}_3 - \vec{\alpha}_3 \vec{\beta}_2)\right\} \quad (2.12)
\end{aligned}$$

Comparing the two-point and the three-point correlators, the irregular ansatz (1.9) solves the OSFT equation of motion provided that the wavefunction $T(\vec{\alpha}, \vec{\beta})$ satisfy the Euler-Lagrange equation following from the cubic nonlocal effective action:

$$S = - \int d^D x d^D y \{ T(x, y) e^{-\square_y} \left(-\frac{1}{2}\square_x - 1\right) T(x, y) + \tilde{\star}\{\tau^3(x, y)\} \} \quad (2.13)$$

where

$$\tau(x, y) = e^{-\frac{43}{27}\square_y} T(x, y) \quad (2.14)$$

is a new (nonlocal) field variable, familiar from rolling tachyon cosmology and the star product with the tilde is defined according to

$$\tilde{\star}\{T_1(x, y) \dots T_N(x, y)\} = \lim_{y_1, \dots, y_N \rightarrow y} e^{\sum_{i,j=1; i < j}^N \frac{43}{27} \partial_{y_i} \partial_{y_j}} T(x, y_1) \dots T(x, y_N) \quad (2.15)$$

This defines the analytic open string field theory solution in terms of rank one irregular vertex operators, generating the higher-spin vertices on the leading Regge trajectory. The generating wavefunction for higher spins is thus described, in the leading order, by the nonlocal action (2.13). The actions of the type (2.13) are well known, as they describe extensions of rolling tachyon dynamics [22, 23], relevant to cosmological models with phantom fields. The nonlocality coefficients appearing in the analytic solution (1.9), (2.13) must be related to cosmological parameters of these models, such as dark energy state parameter and the vacuum expectation values of the rolling tachyon in the equilibrium limit (with the SFT

solution interpolating between two vacua, describing the one dressed tachyon's value $\tau \sim e^{const \square} T$ evolving into the vacuum state satisfying the Sen's conjecture constraints [27, 29]. The solution (1.9), (2.13) also defines the deformations of the BRST charge; solving the OSFT equations with the deformed charge would then result in quartic and higher order corrections in τ . In the commutative level, nonlocal cosmological models of that type have been considered in a number of works (e.g. see [24, 25, 26, 30]). In the case $\vec{\beta} = 0$ (the regular case with the higher spins decoupled) the solution (1.9), (2.13) simplifies and is described by the local cubic action, which is just the leading order low-energy effective action for a tachyon in string perturbation theory. The solution (1.9), (2.13) is then the elementary one, describing the perturbative background deformation of flat target space in the leading order of the tachyon's β -function. With β -parameter switched on, the higher-spin dynamics enters the game and the effective action becomes non-local, describing *non – perturbative* background deformation in open string field theory. The rank one solution, considered so far, can be understood as the one describing generating wavefunction for higher-spin operators on the leading trajectory. It is then straightforward to extend this computation to describe the SFT solutions involving the irregular blocks of higher ranks, generating the higher-spin vertices on arbitrary Regge trajectories. The the effective action describing the generating higher-spin wavefunction essentially remains the same: in the leading order, it is cubic in $\tau = e^{\sum_{j=1}^q a_j \square_{\beta_j}} T(\vec{\alpha}, \vec{\beta}_1, \dots, \vec{\beta}_q)$ (a_j are the constants defining the OSFT solution) with the structure

$$\sim \int d\alpha \prod_j d\beta_j e^{\sum_{j=1}^q b_j \square_{\beta_j}} T\left(\frac{1}{2}\alpha^2 - 1\right) T + \tilde{\star}(\tau^3) \quad (2.16)$$

where τ is again related to T through nonlocal field redefinition.

All the family of the effective actions for collective higher-spin wavefunctions is essentially nonlocal. Clearly, they must be related to the nonlocalities and the star products appearing in higher-spin theories and Vasiliev's equations. It is also remarkable that the generating wavefunction for higher spin fields thus emerges in the context of the rolling tachyon cosmology. Since the solutions of the type (1.9), (2.13) generally describe the nonperturbative deformation of the flat background to collective higher-spin vacuum, it is a profound question whether such a deformation, related to cosmological evolution of generalized rolling tachyon type objects, is subject to constraints set up by the Sen's conjecture [27, 29].

3 Conclusion and Discussion

In this work we have described simple analytic solution in open string field theory, expressed in terms of vertex operators for irregular conformal blocks in the free limit of Toda theory, or bosonic string theory. The wavefunctions $T(\vec{\alpha}; \{\vec{\beta}\})$ for these vertex operators are naturally related to those for the higher-spin vertex operators in open string theory according to

$$H_{\mu_1 \dots \mu_s}(\vec{\alpha}) \sim (-1)^s \prod_{j=1}^s \frac{\delta^{n_j}}{\delta \beta_{n_j}^{\mu_j}} T(\vec{\alpha}; \{\vec{\beta}\}) \delta(\{\vec{\beta}\}) \quad (3.1)$$

We found that the irregular vertex operators analytically solve the equations of open string field theory, provided that the generating wavefunctions satisfy the constraints following from the cubic effective action for generalized rolling tachyons. This implies that the “engineering” of nonperturbative higher-spin vacuum (the nonperturbative deformation of the background from flat to the one described by the minimum of collective higher-spin action, computed to all orders) can be mimicked by the evolution of a rolling tachyon interpolating between inequivalent vacua in cosmological models for the dark energy, with the nonlocality introduced in order to approach the “Big Rip” problem that occurs when the equation of state parameter σ in the equation of state $p = \sigma\rho$ (with p and ρ being the pressure and the energy densities) is less than -1 . To understand the interplays between higher spin dynamics and cosmological models, it shall be important to establish the explicit form of the OSFT solution for higher irregular ranks, in order to include the higher spins on subleading trajectories, and to analyze the equations of motions for the generating wavefunctions $T(\vec{\alpha}, \{\vec{\beta}\})$. We hope to elaborate on this in the future work, currently in progress.

As the β -parameters entering the generating wavefunctions are related to the higher-spin couplings, it is also worth commenting their physical meaning in the context of global space-time symmetries. The α -parameter is clearly related to the momentum of the regular part of the vertex operators. Since $L_0 U_N = \frac{1}{2}(\alpha^2 + \dots)U_N$, the Virasoro generator $L_0 = -\frac{1}{2} \oint dz z P_m^2$, where $P_m = \partial X_m$ is the current for space-time translations, is obviously a quadratic Casimir. As the number of the Toda field (target space) components increases, so does the number of Casimir operators of the space-time symmetry algebra, as well as the highest possible ranks of W_n currents with the irregular vertices being their eigenvectors. Roughly, the highest W_n rank grows with the number Casimirs, but this correspondence is in not an exact match for $n > 4$ and rather subtle, related to an unsolved problem in number theory regarding the calculation of the number of ordered partitions of a given length - in general, the rank of W_n grows faster than the rank of space-time symmetry algebra. Does this imply the presence of hidden dimensions in the theory? In case of the rank one, the single $\vec{\beta}$ -parameter also seems to have an interesting relation to the T -duality transformations in the double field theory context, suggested by orthogonality relation (2.8). Indeed, suppose ϕ is a target space coordinate, compactified on a circle. Then $\partial\phi$ is an operator for an infinitesimal the radius change, while $e^{\beta\partial\phi}$ would define the finite deformations of the compactified dimension. This would in turn define the T -duality transformation with the compactification radius $R \sim \beta^{-1} + \sqrt{4 + \beta^{-2}}$. The explicit construction of vertex operators in double string theories could then be realized in terms of operator algebras involving irregular operators acting on regular states. We hope to address this issue, as well as those outlined above, in the future works.

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